5-person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper for each numbered problem**. Place your team letter in the upper right corner of each page that will be turned in (failure to do this will result in no score). Place problem numbers in the upper left corner. Problems are equally

3. The Kenton County Board of Commissioners, which has 20 members, recently had to elect a President. [redacted]

to be listed in order of preference, with no abstentions. It was found that 11 members (a majority?) preferred A over B (thus the other 9 preferred B over A). Likewise it was determined that 12 members preferred C over A. Given these results, it was suggested that B should withdraw to enable a runoff election [redacted]

1. Let abcd be any four-digit number. Form the 8-digit number n=abcdabcd. Show this number is always divisible by 73.

a. Show that any two-digit number aa, a≠0, is divisible by 11.

c. Finally, show that any 8-digit number abcdabcd, with d≠0, is divisible by 73.

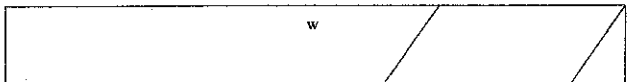
Solution :

a. aa= 10 a + a = (10+1) a = 11 a

b. abcabc=abc(1000)+abc(1)=abc(1001)=abc(13 x 17)= (17abc)(13)

c. abcdabcd=abcd(10000+1)= abcd(10001)=abcd(137)(73)

2. A square is divided into three pieces of equal area by two parallel cuts, as shown. The distance between the parallel lines is 6 cm.
What is the area of the square?



but since each of the three pieces has 1/3 of the total area,

area of a triangle = 1/3 w^2.

Setting these two expressions for the area of a triangle equal and simplifying, we get w(w-3x)=0, and so w=3 x.

The side lengths of the small triangle are x, 6 and $\sqrt{x^2-36}$. Using similar triangles we get $\frac{w}{w-x} = \frac{6}{\sqrt{x^2-36}}$.

Plug 3x in for w in the similar triangles identity and simplify to get $x = \sqrt{52}$ and so

$w = 3\sqrt{52}$. So the area is $9(52)=468$ square centimeters.

3. The Kenton County Board of Commissioners, which has 20 members, recently had to elect a President. There were 3 candidates A (Alice), B (Bob), and C (Carol). On each ballot the 3 candidates were to be listed in order of preference, with no abstentions. It was found that 11 members (a majority ?) preferred A over B (thus the other 9 preferred B over A). Similarly it was found that 12 members preferred C over A. Given these results, it was suggested that B should withdraw to enable a runoff election between A and C. However, B protested, and it was then found that 14 members preferred B over C! The board has not yet recovered from resulting confusion. Given that every possible order of A,B,C appeared on at least one ballot, how many board members voted for B as their fist choice? How about

A and C?

Solution : We use notation in the follwoing way: 7 A > B will mean that 7 voters prefered A to B

three pairs of interest:

3 C > A > B	C>A, A>B
5 A > B > C	A>B,B>C
6 B > C > A	B>C, C>A
1 A > C > B	A>B
1 B > A > C	B>C
1 C > B > A	C>A

Since we need to change a total of 6 pairs, adding to any of the last 3 preferences won't work. But note that we cannot add two more to any one of the first 3 preferences as that will increase 2 of the pairs by 2, and trying to increase the remaining preference will also increase another, making it too large. So

A and 5 C)

than with the one 12-sided die.

(Problem can also be done using probability computations.)

Note : Several teams pointed out that the expected roll value for the two 6-sided dice (7) is higher that that of the one 12-sided die (6.5). While this is true, and would make you suspect that

Caution: Be careful not to make up your own log properties. These are important - learn them cold!